RAN-2203000206020032

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## T.Y.B.Sc. Mathematics (Sem. - VI) Examination September - 2025

## MTH-602 : Linear Algebra - II

**Time: 2 Hours ]** **[ Total Marks: 50**

| સૂચના : / Instructions | Seat No.: |
|---|---|
| (1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book<br>(2) All questions are compulsory.<br>(3) Figures to the right indicate marks of the questions.<br>(4) Follow usual notations. | Student's Signature |

**Q. 1. Answer the following questions (Any Five).** **(10)**

(1) State existence theorem of linear transformation T : $U \to V$.

(2) Let T : $V_2 \to V_2$ be a linear map define by $T(x, y) = (x, y^2)$. Is T a 1-1 linear map? Justify your answer.

(3) Find the linear map associate with a matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ relative to both standard basis.

(4) Let T : $V_3 \to V_2$ define by T $(e_1) = (2, 1)$, T $(e_2) = (3, 2)$, T $(e_3) = (1, 1)$ then find Null space.

(5) Find the range and rank of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

(6) Prove that : A linear transformation T : $V_2 \to V_2$ is an on - to then $N(T) = \{\theta_{V_2}\}$.

(7) Orthonormalized the L. I set $\{(0, 1), (-1, 1)\}$ by Gram Schmidt's process.

(8) Is the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ non-singular ? Justify your answer.

**Q. 2. Answer the following (Any two).** **(10)**

(1) Let $T : U \to V$ be a one - one linear map and $u_1, u_2, u_3, \ldots\ldots u_n$ are L. I vectors of $U$ then prove that $T(u_1), T(u_2), T(u_3), \ldots.. T(u_n)$ are L. I vectors in $R(T)$.

(2) Let $T : U \to V$ be a linear map. Prove that the range $R(T)$ is a subspace of a vector space $V$.

(3) Obtain the general rule for the given linear transformation :
$T : V_3 \to V_2$ defined by $T(e_1) = 2f_1 - f_2$, $T(e_2) = f_1 + 2f_2$, $T(e_3) = 0f_1 + 0f_2$ Where $B_1 = \{e_1, e_2, e_3\}$ and $B_2 = \{f_1, f_2\}$ are standard basis.

**Q. 3. Answer the following (Any two). (10)**

(1) Verify Rank - Nullity theorem for a linear transformation T : $V_3 \to V_3$ such that
$T : V_3 \to V_3$ ; $T(e_1) = e_1 + 2e_2$ , $T(e_2) = 2e_2 + e_3$ , $T(e_3) = e_3 + 2e_1$ .

(2) Let $S : V \to W$ and $T : U \to V$ be two linear map then prove that

(a) $ST$ is one-one then $T$ is one-one

(b) $ST$ is on-to then $S$ is on-to.

(3) Find the inverse linear transformation of the given linear transformation $T$ if it exists.
$T : V_3 \to V_3$ ; $T(e_1) = e_1 + e_2$ , $T(e_2) = e_2 + e_3$ , $T(e_3) = e_1 + e_2 + e_3$.

**Q. 4. Answer the following (Any two). (10)**

(1) Find the matrix $(T ; B_1 , B_2)$ associated with a linear transformation $T : V_2 \to V_2$ defined by $T(x, y) = (x, -y)$ relative to basis $B_1 = \{(1, 1), (1, 0)\}$ and $B_2 = \{(2, 3), (4, 5)\}$.

(2) Verify the Rank - Nullity theorem for the matrix $\begin{bmatrix} 2 & 4 & 3 \\ -2 & 8 & 3 \\ 2 & 0 & 1 \end{bmatrix}$

(3) Find the Linear transformation $T$ associated with a matrix $\begin{bmatrix} 2 & 1/2 & 3/2 \\ 1 & 3/2 & 1/2 \end{bmatrix}$

relative to basis $B_1 = \{(1,1,0), (1,0,1), (0,1,1)\}$ and $B_2 = \{(1,1), (1, -1)\}$.

**Q. 5. Answer the following (Any two). (10)**

(1) In an Inner product space $V$, prove that $\| u \cdot v \| \leq \| u \| \, \| v \|$; $u, v \in V$.

(2) In an Inner Product space $V$, prove that Any orthogonal set of non zero vectors is always L. I.

(3) Orthonormalized the L. I set $\{(1, 2, 1), (1, -1, 0), (5, -1, 2)\}$ by Gram Schmidt's process.